Readers' Forum

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Comment on "Induced Drag Based on Leading-Edge Suction for a Helicopter in Forward Flight"

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THE authors of Ref. 1 have tackled an important problem, namely, the prediction of the induced drag and required power on helicopter rotors in forward flight. However, the authors assertion that "lifting-line theory is not adequate in describing helicopters in forward flight because the flowfield is unsteady; there are multiblade interference effects, and the wake is curved" is rather misleading. It is the purpose of this Comment to provide some additional clarification of the issue of induced drag prediction on rotors.

It is generally well recognized that all the above-quoted effects can be modeled, albeit approximately, within the confines of a lifting-line/blade element analysis of a rotor in forward flight (see, for example, Ref. 2). The limitations of lifting-line theory are well known for low aspect ratio and/or swept wings, however, the shortcomings of conventional lifting-line theory (or induced angle model) as a tool for computing the induced power on the relatively high aspect ratio blades found on helicopter rotors are not as severe as the authors of Ref. 1 portray. In fact, the results from this type of analysis have been shown to give quite satisfactory predictions of rotor loads and performance, including induced power. Certainly, induced power remains one of the most difficult aspects of the rotor performance to predict; however, even classical momentum theory (and empirical modifications thereof) will do a reasonable job in predicting the net induced power of most conventional rotors.

The main issue of concern to rotor aerodynamicists is not the inadequacies of lifting-line theory, but the wake models used to compute the induced inflow (i.e., free wakes vs prescribed wakes), since it is the wake model that largely determines the accuracy of the predictions of induced angle of attack at the blade sections, and hence the induced drag. Nevertheless, it is acknowledged that problems can occur with lifting-line theory, mainly when modeling rotor blades with complex tip planforms, which may involve both changes in sweep angle and chord. For highly swept tips, lifting-line theory can become increasingly inaccurate for both spanwise lift and induced power predictions. Consequently, it is preferable that lifting-surface theory be used, particularly for forward flight calculations. However, this still does not preclude the use of a lifting-line theory (or induced angle model), since this theory has also been extended to rotors with swept tips.[3]

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It is difficult for the reader of Ref. 1 to understand why the induced drag results computed by the induced angle model vs the "leading-edge suction model" (as given in the authors' Table 1) are so vastly different, even on a conventional, unswept, untapered rotor blade. It is even more surprising to find that in Fig. 1 of Ref. 1 the "leading-edge suction model" agrees so well with experimental measurements, even though these results are for hover, not forward, flight. Unfortunately, there is no breakdown of the relative contributions due to induced vs profile power, or comparisons with other theories such as Refs. 4 or 5, which could be used to clarify the results in the authors' Fig. 1 and thereby more clearly illustrate the claimed benefits of the "leading-edge suction model."

Assuming the comparison was performed on an equal thrust basis, then the fact that the induced drag predictions in Table 1 of Ref. 1 are so uniformly different between the two methods is surely of some major concern. Clearly, it might be expected that the methods may differ at the root and tip of the blade where three-dimensional effects prevail, but over the midspan of the blade the results should be in good agreement. Furthermore, lifting-line theory tends to overpredict the induced drag compared with the more accurate lifting-surface theory. Thus, the trends shown in Table 1 are inconsistent with what might be expected from a more accurate model.

Since the induced drag distribution over the blade in forward flight will also be a function of blade azimuth, it is not clear from Table 1 what azimuth angle is being used for the comparison. Also, it is not clear how "Theodorsen's theory" can be applied to these results, if at all. Besides the fact that the "reduced frequency" on a rotor in forward flight is an ambiguous quantity, unsteady effects will contribute to both C_n and C_s in a manner that cannot be accounted for by an arbitrary application of Theodorsen's theory.

As far as the concept of leading-edge suction is concerned, there is a fairly large amount of literature on this topic, e.g., Refs. 6-10. However, for any finite wing, one must question the validity (and unusual form) of Eq. (3) in Ref. 1. In the three-dimensional rotor case, the suction force depends critically on the actual induced velocity u at the leading edge of the blade, which is affected by both blade planform *and* wake effects. The suction and corresponding induced drag (and hence induced power) cannot be predicted more accurately by a simple scaling modification of the two-dimensional result, as implied by Eqs. (1-6) in Ref. 1.

In linearized compressible flow, the leading-edge suction coefficient C_s at a particular section on a thin wing with leading-edge sweep angle Λ can be developed through the concepts given in Ref. 11, and can be written in the form

$$C_s = (\pi/8)\sqrt{(\beta^2 + \tan^2 \Lambda)}\lim_{\xi \to 0}[\xi \Delta C_p(\xi)^2] \tag{1}$$

where β is the Prandtl-Glauert factor and ΔC_p is the differential pressure coefficient at the wing section, as obtained from any lifting-surface theory. The parameter ξ is the distance aft from the leading edge of the wing, nondimensionalized by local chord.

As a particular example, a thin unswept two-dimensional wing at an angle of attack α in a subsonic flow has a chordwise load distribution given exactly by the expression

$$\Delta C_p(\xi) = (4\alpha/\beta)\sqrt{1 - \xi/\xi} \tag{2}$$

and, therefore, the leading-edge suction for this case is simply

$$C_s = (\pi\beta/8)\lim_{\xi\to 0} [(16\alpha^2/\beta^2)(1-\xi)] = (2\pi/\beta)\alpha^2 \tag{3}$$

The corresponding pressure drag is obtained by resolving the components of the sectional normal force C_n and the leading-edge suction force C_s through the (geometric) angle of attack α, which is given to a small angle approximation by Eq. (4) of Ref. 1, viz.,

$$C_d = C_n\alpha - C_s \tag{4}$$

Substitution of the two-dimensional results gives the well-known result of the drag on a two-dimensional thin airfoil in a steady inviscid flow being exactly zero. The effects of three-dimensionality, viscosity, and unsteadiness will, however, modify this result.

In three-dimensional flow, such as a finite wing or helicopter rotor blade, the pressure distribution is no longer given in the form of Eq. (2) [or Eq. (1) of Ref. 1] and must be derived numerically for the particular planform in question, using lifting-surface theory. The usual approach for rotor blades generally takes the form of a vortex lattice method or an equivalent doublet panel method, such as Ref. 12. Based on the geometry of the blades, aerodynamic influence coefficients are formulated that relate the doublet strengths to the downwash at the rotor control points. These influence coefficients form a linear system of equations, the doublet strengths being uniquely defined by the boundary condition of flow tangency on the rotor camber surfaces as well as by the Kutta-Joukowski condition along the trailing edges of the blades.

The wake contribution can be modeled using a discretized helical wake for hovering rotors, or a discretized cycloidal wake in forward flight. More realistic wake geometries can be used based on either prescribed or free-wake calculations, such as discussed in Refs. 2 or 12. Compressible flow effects can also be introduced using Goethert type transformations to the rotor and the wake geometry. The resulting doublet strengths provide the velocity perturbation potential $\Phi(\xi,\eta)$, where the variable η represents the radial/spanwise coordinate on the blade. The chordwise velocity and pressure distributions are subsequently derived from the gradient of this perturbation potential. This overall form of calculation is fairly routine, even for the somewhat more lengthy analysis typically associated with helicopter rotor blades in forward flight.

In the rotor case, the leading-edge suction force is still given by the limiting form of Eq. (1), however, it should be noted that the suction force (and corresponding induced drag) depends critically on $\lim_{\xi\to 0}\xi\Delta C_p^2(\xi,\eta)$ across the leading-edge part of the blade. Because of the induced effects (both from the blade planform and the wake), the forward component of suction force no longer balances the rearward component of the normal force and a net "induced drag" is obtained. This drag is usually positive on wings, but can actually be negative (a propulsive force) on a rotor blade over the front of the rotor disk in forward flight, during a descent, or in autorotative flight.

Of course, in the three-dimensional case the definitive value obtained for the leading-edge suction is quite sensitive to the accuracy to which Φ and its derivative, u (or C_p), is calculated. Furthermore, by virtue of the discretized process by which Φ itself is normally calculated, the associated evaluation of $\lim_{\xi\to 0} [\xi\Delta C_p(\xi,\eta)^2]$ at the leading edge becomes difficult, and special numerical extrapolation techniques must normally be employed (see, for example, Ref. 10). However, the main point here is that by assuming the two-dimensional form of chordwise pressure, as presumed in Ref. 1, the induced drag on a rotor blade cannot be predicted more accurately than by the conventional induced angle model.

It is also important to note that the induced drag on either a two- or three-dimensional lifting surface is certainly affected by unsteady effects. For example, it is well known that the drag on a thin airfoil in steady inviscid flow is zero, but this is not true for unsteady inviscid flow. When unsteady effects are present, such as on a helicopter rotor in forward flight, there are additional contributions to the chordwise pressure loading due to terms that have their origin from Kelvin's equation (or unsteady Bernoulli equation), as well as from the shed wake. Based on the thin airfoil assumptions invoked in lifting surface theory, the $\partial\Phi/\partial t$ terms do not contribute to the unsteady suction force, however the induced drag will still be affected by unsteadiness because of the wake effects on C_s and C_n, as well as the effect of the $\partial\Phi/\partial t$ terms on C_n. Hence, unsteady effects on the drag must appear through Eq. (4). These additional unsteady effects to the sectional drag may also be included in a lifting-line/blade element model of the rotor using a shed wake theory and the equivalent apparent mass contributions to C_n.

By way of completeness in this Comment, it is important to note that in many practical applications of lifting-surface theory (both steady and unsteady) to rotors, the approximate effects of viscosity can be introduced into the theory for the profile drag through the use of a leading-edge suction recovery factor ϵ. For example, in the simple two-dimensional case the pressure drag will be given by

$$C_d = (2\pi/\beta)(1-\epsilon)\alpha^2 \tag{5}$$

Values for ϵ are typically about 0.95, and can be derived empirically from available two-dimensional airfoil drag measurements, as required. The contributions due to viscous shear are additive, of course, and will be required in order to accurately predict the total rotor power requirements (induced plus profile) in hover or forward flight. Since these latter effects appear not be included in the theory of Ref. 1, the almost perfect agreement with test data, as shown by their Fig. 1, remains rather surprising.

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